Stabilization of Autonomous Bicycle using Fuzzy Controller with Maximum Allowable Lean Constraint

Himanshu Dutt Sharma[1] and UmaShankar N[2]

[1]Embedded Systems Lab, Central Electronic Engineering Research Institute (CEERI), Pilani- 333031, Rajasthan, INDIA

Email: hdsharma@gmail.com / hdsharma@ceeri.ernet.in

[2]Department of Mechanical Engineering, Birla Institute of Technology and Science (BITS), Pilani- 333031, Rajasthan, INDIA

Email: f2002901@bits-pilani.ac.in

Abstract

The authors have developed an intelligent control system for stabilizing autonomously running bicycle by controlling its lean alone. The controller is developed using fuzzy logic approach with a maximum allowable lean of ± 0.05 radians i.e. ± 2.87º wherein the rule set is designed using the inherent-characteristic relationship of lean and steer present in a bicycle. The Lagrangian mechanics based bicycle model along with the controller is simulated in MATLAB and the results confirm that the controller effort successfully stabilizes the bicycle at various velocities.

1 Introduction

BICYCLES have been a very common and efficient means of transportation and recreation [1]. But they have an interesting non-trivial behavior with reference to Dynamic Stability [2]. A Basic Bicycle consists of four linked rigid bodies, namely a rigid frame with a fixed rigid driver, front fork/handle bar assembly, and front and rear wheels which have a symmetry plane [3], Figure 2. It has five degrees of freedom in finite motion, but only two such degrees of freedom are present in infinitesimal linear motion [4]. A bicycle is a doubly non-holonomic system [5]. This non-holonomic system is characterized by complex coupled second order differential equations.

Control of the autonomous bicycle is a rich problem offering a number of considerable challenges of current research interest in the area of mechanics and robot control [6, 7]. Fuzzy Logic control has emerged as an alternative or complement to conventional control strategies in many engineering areas, especially in robotics [8]. Fuzzy control theory usually provides nonlinear controllers that are capable of performing various nonlinear control actions [9]. If the parameters of the fuzzy controllers are chosen appropriately, it is also possible for them to work for uncertain nonlinear systems [10]. In addition, fuzzy controllers are capable of handling many complex situations such as some control systems with large uncertainties in process parameters and/or systems structures, as well as some ill-modeled or linguistically described physical systems [11]. In this paper, we design and implement a Fuzzy Lean Controller for stabilizing the Autonomous Bicycle. To validate the effectiveness of the controller under a significant uncertain environment, we simulated the Autonomous Bicycle at various velocities.

2 Mechanics of Bicycle

2.1 Basic Terminologies

The Bicycle model (Figure 2), under consideration, has a rider rigidly fixed to the rear frame of the bicycle and he doesn't control over the bicycle. A bicycle has several components and various design parameters. For understanding these parameters, some basic definitions [3] are given below:

Trail – the distance, measured with the bicycle in an upright position, from the intersection of the steering axis with ground back to the contact point of the front wheel.

Mechanical trail – measured the same as trail, this is the perpendicular distance from the steering axis to the contact point of the front wheel.

Rake (fork rake) – the perpendicular distance from the steering axis to the center of the front wheel.

Tilt of steering axis – the angle defined by the steering axis and the vertical.

Sliding (skidding) – motion of the contact point of wheel relative to the ground.

Sideslip – sideways motion of the contact point of wheel relative to the ground.

Slip angle- the angle made with the tangent to the contact point path (instantaneous velocity direction) and the line from the intersection of the plane of the front wheel and the ground plane.

Lean i.e. tip/roll, *steer, yaw, lateral displacement, and forward displacement* – these are generalized coordinates and auxiliary variables used to describe the motion of the bicycle.The different planes in a bicycle along with their lean and steer angles are shown below in Figure 1.

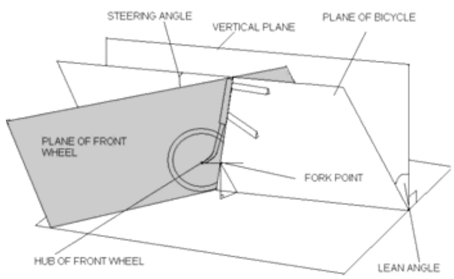

Figure 1: Planes in a Bicycle.

2.2 Bicycle Parameters

The parameters of the bicycle, considered for simulation purposes, are tabulated below (Table 1). These are measured from a bicycle being used in the ongoing experiments on Autonomous Bicycle.

Table 1: Bicycle Parameters

Mass of the bicycle	20 kg
Mass of the front wheel	3.3 kg
Mass of the rear wheel	3.3 kg
Steering Axis Tilt	16º
Height of the bicycle (Measured from handle bar top)	0.93 m
Radius of the wheel rim (front and rear)	0.28 m
Radius of the wheel (front and rear) measured from wheel centre	0.33 m
Trail	0.026 m
Mechanical Trail	0.025 m
Distance between front and rear wheels' ground contact points	1.07 m

2.3 Equations Governing Motion of a Bicycle

For the analysis and experiments conducted on basic bicycle model in simulation, the governing equations [3] are used with following *assumptions*:

-A Basic bicycle consists of four rigid bodies: Rear frame with fixed rider, Front fork / handle bar assembly, Front wheel, Rear wheel (Figure 2).

-Bicycle rider system is symmetric about vertical plane passing length wise through the middle of the rear plane.

-The wheels are rotationally symmetric about their axles.

-The rider does not move relative to the frame. This case is called as passive rider analysis

-The tires are a part of wheels which are considered as rigid disks. Also the tires are infinitely stiff, and side slip angle is zero in all cases.

-Within the bicycle-rider system, there is no friction or propulsion acting on the wheels i.e. there is no friction or bearing/pedaling torques between the wheels and axles or frame.

-The bicycle wheels roll on a rigid flat horizontal surface with enough friction between the wheels and the road to prevent sliding.

-Air drag is neglected.

-Only small disturbances from the vertical straight ahead equilibrium motion position are considered.

-The lateral load at the wheel contacts is just that required to cause no side slip between the wheel and the ground.

-The bicycle travels with a constant velocity, which is a consequence of the linearized equations.

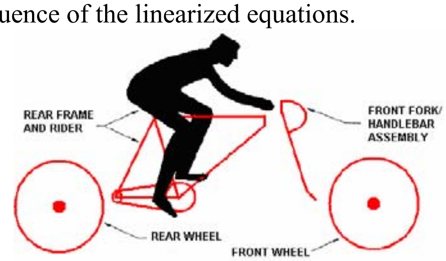

Figure 2: Four Rigid Bodies of the Bicycle.

The solution to the Lagrangian Equation for all the generalized co-ordinates results into two second order coupled linear differential equations which correspond to the Lean (χ) and Steer (ψ) coordinates (Equations 1 and 2). The Lean and Steer Equations are:

Lean Equation:

$$M_{\chi\chi}\,\ddot{\chi}_r + K_{\chi\chi}\,\chi_r + M_{\chi\psi}\,\ddot{\psi} + C_{\chi\psi}\,\dot{\psi} + K_{\chi\psi}\,\psi = 0 \quad (1)$$

Steer Equation:

$$M_{\psi\psi}\,\ddot{\psi} + C_{\psi\psi}\,\dot{\psi} + K_{\psi\psi}\,\psi + M_{\psi\chi}\,\ddot{\chi}_r + C_{\psi\chi}\,\dot{\chi}_r + K_{\psi\chi}\,\chi_r = 0 \quad (2)$$

The formulae of the terms used in these equations are given in the Appendix.

3 Design of Fuzzy Lean Controller

In the previous section, the problem and mechanics of the bicycle have been described. This section aims at designing the fuzzy controller. The design of the fuzzy sets is generally done based on experience and observation of the designer, and according to the needs of the system.

The fuzzy lean controller for the bicycle is designed with two inputs and one output. The inputs are "Current

Lean” and “Current Steer”, while the output is “Lean Correction”. The controller aims at controlling only lean so as to maintain stability. This is done making use of the inherent relationship between lean and steer.

3.1 Fuzzy Membership Functions

The fuzzy lean controller is designed to control the bicycle in a prescribed range of lean and steer. The possible values for current lean are -30º to +30º, while the possible values for current steer and lean correction are -45º to +45º and -40º to +40º respectively.

Therefore, we have divided the fuzzy input – current lean set as follows: (Figure 3)

--Very Small Positive (VSP) (0º to 3º)
--Small Positive (SP) (0º to 5º)
--Medium Positive (MP) (3º to 10º)
--Large Positive (LP) (5º to 15º)
--Very Large Positive (VLP) (10º to 30º)

Similarly for the negative side:

--Very Small Negative (VSN) (-3º to 0º)
--Small Negative (SN) (-5º to 0º)
--Medium Negative (MN) (-10º to -3º)
--Large Negative (LN) (-15º to -5º)
--Very Large Negative (VLN) (-30º to -10º)

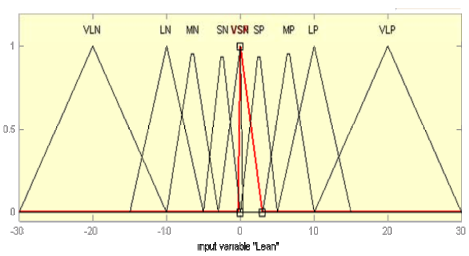

Figure 3: Membership Functions of Input Lean

Further, we have divided the fuzzy input – current steer set as follows: (Figure 4)

--Very Small Positive (VSP) (0º to 5º)
--Small Positive (SP) (0º to 10º)
--Medium Positive (MP) (5º to 20º)
--Large Positive (LP) (10º to 30º)
--Very Large Positive (VLP) (20º to 45º)

Similarly for the negative side:

--Very Small Negative (VSN) (-5º to 0º)
--Small Negative (SN) (-10º to 0º)
--Medium Negative (MN) (-20º to -5º)
--Large Negative (LN) (-30º to -10º)
--Very Large Negative (VLN) (-45º to -20º)

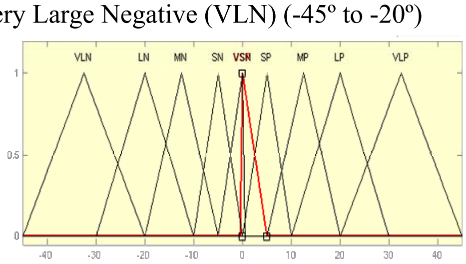

Figure 4: Membership Functions of Input Steer

Further, we have divided the fuzzy output – lean correction set as follows: (Figure 5)

--Very Small Positive (VSP) (0º to 5º)
--Small Positive (SP) (0º to 10º)
--Medium Positive (MP) (5º to 15º)
--Large Positive (LP) (10º to 25º)
--Very Large Positive (VLP) (15º to 40º)

Similarly for the negative side:

--Very Small Negative (VSN) (-5º to 0º)
--Small Negative (SN) (-10º to 0º)
--Medium Negative (MN) (-15º to -5º)
--Large Negative (LN) (-25º to -10º)
--Very Large Negative (VLN) (-40º to -15º)

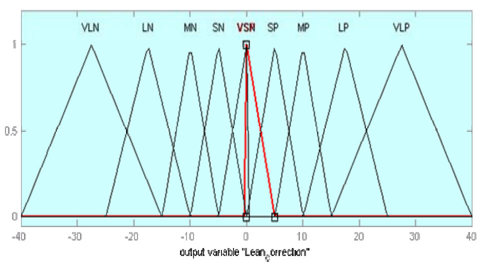

Figure 5: Membership Functions of Output Lean Correction

3.2 Fuzzy Rules

With the designed fuzzy sets, we have defined the rules needed to develop the fuzzy lean controller for the bicycle. The rules have been designed on the basis of complete practical understanding of the relationship between lean and steer. The developed fuzzy rules are given below.

1) IF the LEAN is VSP AND the STEER is VSN THEN the Correction is VSN.
2) IF the LEAN is SP AND the STEER is VSN THEN the Correction is SN.
3) IF the LEAN is MP AND the STEER is VSN THEN the Correction is MN.
4) IF the LEAN is LP AND the STEER is VSN THEN the Correction is MN.
5) IF the LEAN is VLP AND the STEER is VSN THEN the Correction is LN.
6) IF the LEAN is VSP AND the STEER is SN THEN the Correction is SN.
7) IF the LEAN is SP AND the STEER is SN THEN the Correction is SN.
8) IF the LEAN is MP AND the STEER is SN THEN the Correction is MN.
9) IF the LEAN is LP AND the STEER is SN THEN the Correction is LN.
10) IF the LEAN is VLP AND the STEER is SN THEN the Correction is VLN.
11) IF the LEAN is VSP AND the STEER is MN THEN the Correction is SN.

12) IF the LEAN is SP AND the STEER is MN THEN the Correction is MN.
13) IF the LEAN is MP AND the STEER is MN THEN the Correction is MN.
14) IF the LEAN is LP AND the STEER is MN THEN the Correction is LN.
15) IF the LEAN is VLP AND the STEER is MN THEN the Correction is VLN.
16) IF the LEAN is VSP AND the STEER is LN THEN the Correction is MN.
17) IF the LEAN is SP AND the STEER is LN THEN the Correction is MN.
18) IF the LEAN is MP AND the STEER is LN THEN the Correction is LN.
19) IF the LEAN is LP AND the STEER is LN THEN the Correction is LN.
20) IF the LEAN is VLP AND the STEER is LN THEN the Correction is VLN.
21) IF the LEAN is VSP AND the STEER is VLN THEN the Correction is MN.
22) IF the LEAN is SP AND the STEER is VLN THEN the Correction is LN.
23) IF the LEAN is MP AND the STEER is VLN THEN the Correction is LN.
24) IF the LEAN is LP AND the STEER is VLN THEN the Correction is VLN.
25) IF the LEAN is VLP AND the STEER is VLN THEN the Correction is VLN.
26) IF the LEAN is VSN AND the STEER is VSP THEN the Correction is VSP.
27) IF the LEAN is SN AND the STEER is VSP THEN the Correction is SP.
28) IF the LEAN is MN AND the STEER is VSP THEN the Correction is MP.
29) IF the LEAN is LN AND the STEER is VSP THEN the Correction is MP.
30) IF the LEAN is VLN AND the STEER is VSP THEN the Correction is LP.
31) IF the LEAN is VSN AND the STEER is SP THEN the Correction is SP.
32) IF the LEAN is SN AND the STEER is SP THEN the Correction is SP.
33) IF the LEAN is MN AND the STEER is SP THEN the Correction is MP.
34) IF the LEAN is LN AND the STEER is SP THEN the Correction is LP.
35) IF the LEAN is VLN AND the STEER is SP THEN the Correction is VLP.
36) IF the LEAN is VSN AND the STEER is MP THEN the Correction is SP.
37) IF the LEAN is SN AND the STEER is MP THEN the Correction is MP.
38) IF the LEAN is MN AND the STEER is MP THEN the Correction is MP.
39) IF the LEAN is LN AND the STEER is MP THEN the Correction is LP.
40) IF the LEAN is VLN AND the STEER is MP THEN the Correction is VLP.
41) IF the LEAN is VSN AND the STEER is LP THEN the Correction is MP.
42) IF the LEAN is SN AND the STEER is LP THEN the Correction is MP.
43) IF the LEAN is MN AND the STEER is LP THEN the Correction is LP.
44) IF the LEAN is LN AND the STEER is LP THEN the Correction is LP.
45) IF the LEAN is VLN AND the STEER is LP THEN the Correction is VLP.
46) IF the LEAN is VSN AND the STEER is VLP THEN the Correction is MP.
47) IF the LEAN is SN AND the STEER is VLP THEN the Correction is LP.
48) IF the LEAN is MN AND the STEER is VLP THEN the Correction is LP.
49) IF the LEAN is LN AND the STEER is VLP THEN the Correction is VLP.
50) IF the LEAN is VLN AND the STEER is VLP THEN the Correction is VLP.

3.3 Defuzzification

Defuzzification refers to the way a crisp value is extracted from a fuzzy set as a representative value. The defuzzified output is obtained by using the above fuzzy sets and fuzzy rules. The ‘centroid’ method is used as the defuzzification strategy.

4 Simulation tests and Validation of the Controller

The block diagram of the fuzzy controller is given in Figure 6. Using equations (1 & 2) of the lagrangian bicycle model, the variation of lean and steer with time can be studied. The effectiveness of the controller is established by validating its control effort for different velocities of the autonomous bicycle. Here, three such constant velocity cases, namely, 5m/s, 10 m/s and 15 m/s are illustrated.

To demonstrate control on highly unstable bicycle system and to verify the fuzzy lean controller a MATLAB code has been written to implement the fuzzy control system. The steps involved in the control are as follows.

The bicycle model is simulated with controller effort without specific time intervals. Between the consecutive efforts of the controller, the model is left to its natural dynamics. The maximum lean that the controller allows the bicycle to reach is ± 0.05 radians (=± 2.87º). The moment the bicycle reaches boundary value, the lean and steer values are fed to the controller and the lean correction is determined. The controller reverses the direction of the lean rate, without change in magnitude i.e. rate continuous and waits until the bicycle reaches the corrected lean position, running autonomously. This position's lean and steer are now fed back to the controller and the lean correction is determined. This process keeps repeating.

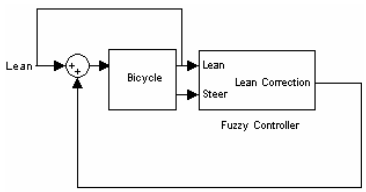

Figure 6: Block Diagram of the Simulation Model and the Fuzzy Control System

Illustration (i): Assuming that the velocity of the bicycle is constant at 5 m/s, the variation of lean and steer with time for the controlled autonomous bicycle is given in Figure 7 and Figure 8 respectively.

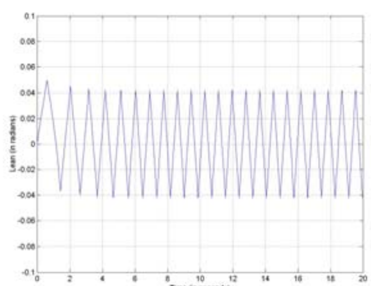

Figure 7: Controlled Lean Variation at 5 m/s

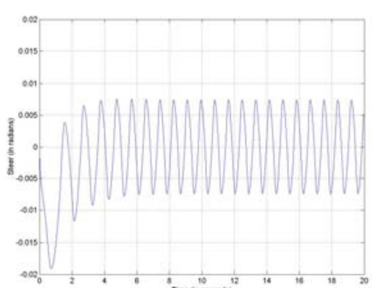

Figure 8: Controlled Steer Variation at 5 m/s

The graphs in Figure 7 and Figure 8 show that the bicycle traveling at an unstable velocity of 5 m/s is controlled by the fuzzy lean controller. It can be noted that, after 7.3465 seconds, the bicycle has become completely stabilized repeatedly reaching lean values of ± 0.0423 radians (= ± 2.42º) and steer values of ±0.0053 radians (=± 0.30º). Hence the fuzzy lean controller stabilizes the bicycle within the range specified for stability i.e. a lean within ± 2.87º. As evident, the steer has extremely small variations in steady state.

Illustration (ii): Assuming that the velocity of the bicycle is constant at 10 m/s, the variation of lean and steer with time for the controlled autonomous bicycle is given in Figure 9 and Figure 10 respectively. The graphs show that the bicycle traveling at a velocity of 10 m/s, stabilizes itself after 13.2951 seconds, as a result of the controller effort.

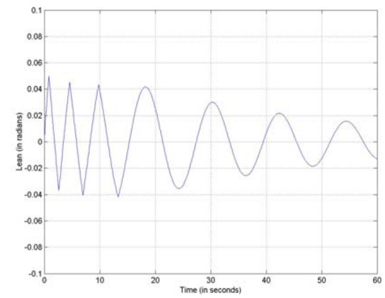

Figure 9: Controlled Lean Variation at 10 m/s

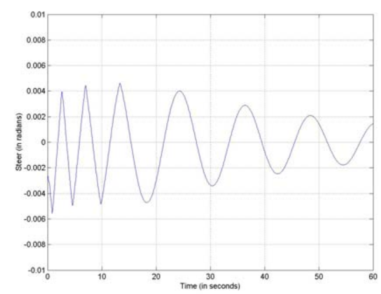

Figure 10: Controlled Steer Variation at 10 m/s

Illustration (iii): Assuming that the velocity of the bicycle is constant at 15 m/s, the variation of lean and steer with time for the controlled autonomous bicycle is given in Figure 11 and Figure 12 respectively.

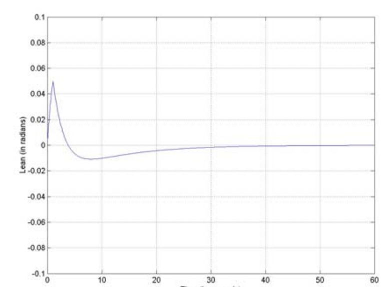

Figure 11: Controlled Lean Variation at 15 m/s

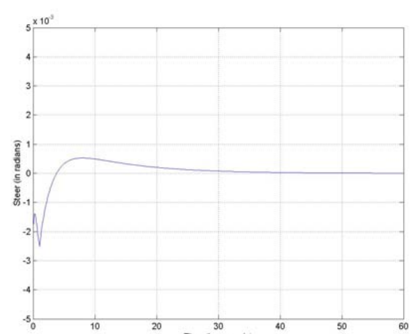

Figure 12: Controlled Steer Variation at 15 m/s

The graphs in Figure 11 and Figure 12 show that the bicycle traveling at a velocity of 15 m/s is controlled by the fuzzy lean controller. It can be noted that, just after a single controller action, the lean and steer of the bicycle become zero after 35 seconds. The non-oscillatory behavior is because it is at the middle of the stable velocity region with real and distinct roots, which cause the lean vary exponentially [12].

5 Conclusions

An autonomous bicycle's equilibrium values for lean and steer are theoretically zero. The aim in developing a controller is to achieve these values and to maintain them. The fuzzy lean controller that we have developed achieves a result that is very close to these equilibrium values at all velocities and reaches this equilibrium value at 15m/s. The bicycle system is inherently less stable at lower speeds and conversely, for higher speeds, it exhibits more inherent stability. This is proved by the fact that the controller is able to stabilize the bicycle in less time spans at higher velocities. It is able to stabilize the autonomous bicycle easily in the stable velocity region and requires considerable effort in the unstable velocity region. The stabilizing steer value decreases with increase in bicycle velocity. This shows that the controller that we have developed is very efficient in controlling the steer as well. It clearly proves that the autonomous bicycle system can be effectively stabilized by just controlling its lean.

6 Appendix

Terms used in Equations of Motion

$$M_{\chi\chi}\ \ddot{\chi}_r + K_{\chi\chi}\ \chi_r + M_{\chi\psi}\ \ddot{\psi} + C_{\chi\psi}\ \dot{\psi} + K_{\chi\psi}\ \psi = 0$$

$$M_{\psi\psi}\ \ddot{\psi} + C_{\psi\psi}\ \dot{\psi} + K_{\psi\psi}\ \psi + M_{\psi\chi}\ \ddot{\chi}_r + C_{\psi\chi}\ \dot{\chi}_r + K_{\psi\chi}\ \chi_r = 0$$

where

$$M_{\chi\chi} = T_{yy}$$

$$C_{\chi\chi} = 0$$

$$K_{\chi\chi} = -g\ m_t\ h_{\bar{t}}$$

$$M_{\chi\psi} = F'_{\lambda y} + \frac{c_f}{c_w} T_{yz}$$

$$C_{\chi\psi} = -\left(H_f\ Cos\lambda + \frac{c_f}{c_w} H_t\right) + V\left(T_{yz}\ \frac{Cos\ \lambda}{c_w} - \frac{c_f}{c_w}\ m_t\ h_{\bar{t}}\right)$$

$$K_{\chi\psi} = g\ v - H_t\ V\ \frac{Cos\ \lambda}{c_w} - V^2\ \frac{Cos\ \lambda}{c_w}\ m_t\ h_{\bar{t}}$$

$$M_{\psi\psi} = F'_{\lambda\lambda} + 2\frac{c_f}{c_w} F''_{\lambda z} + \frac{c_f^{\ 2}}{c_w^{\ 2}} T_{zz}$$

$$K_{\psi\chi} = g\ v$$

$$C_{\psi\psi} = V\left[\frac{Cos\ \lambda}{cw}\left(F''_{\lambda z} + \frac{c_f}{c_w} T_{zz}\right) + \frac{c_f}{c_w}\ v\right]$$

$$K_{\psi\psi} = -g\ v\ Sin\ \lambda + V\ H_f\ \sin\lambda\ \frac{Cos\ \lambda}{c_w} + V^2\ v\ \frac{Cos\ \lambda}{c_w}$$

$$M_{\psi\chi} = F'_{\lambda y} + \frac{c_f}{c_w} T_{yz}$$

$$C_{\psi\chi} = H_f\ Cos\lambda + \frac{c_f}{c_w} H_t$$

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